UNIT 5
TRUTH TABLES FOR TESTING VALIDITY

- Using truth tables for testing validity
  - The partial method
  - The short method

Truth Tables for Testing Validity: An Overview

- Truth tables are systematic representations of **all the truth value combinations** that the variables of a formula(s) can have.

- An important use of truth tables is **testing the validity** of arguments.

1. p ⊃ q
2. q ⊃ p
3. p v q

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4. p • q

p	q	(p ⊃ q)	(q ⊃ p)	(p v q)	/∴ (p • q)
T	T	T	T	T	T
F	T	T	F	T	F
T	F	F	T	T	F
F	F	T	T	F	F

- Truth tables are **mechanical decision procedures**: They will **always give an answer** (valid/invalid)

Testing Validity With Truth Tables

- Main aim: To **find counterexamples** for argument forms.

- A counterexample is **a row in the table on which all premises are true, but conclusion is false**.

- If there is a counterexample, the argument is **invalid**.
- If there is none, the argument is **valid**.

Testing Validity with Truth Tables: An Example

"**If** I live close to school I'll pay a lot in rent, **and if** I **don't** live close to school I'll pay a lot for gas, **so** I'll pay a lot for **either** rent **or** gas"

1. **If** I live close to school I'll pay a lot in rent
2. **If** I don't live close to school I'll pay a lot for gas

/:. 3. I'll pay a lot for **either** rent **or** gas

- I live close to school……... C
- I pay a lot in rent …………. R
- I pay a lot in gas …………… G

Preparatory Step: Formalizing the Argument

- In order to use truth tables, you have to **put the argument in symbolic language** (if it is not already).

- Hence the argument from last slide should become:

Semi-formally:	Formally:
1. **If** C, **then** R	1. (C ⊃ R)
2. **If not** C, **then** G	2. (~C ⊃ G)
/:. 3. R **or** G	/:. 3. (R v G)

A Subtle Point

Strictly speaking, what is evaluated with the truth tables is **the form** of the argument.

Argument instance:

1. (C ⊃ R)
2. (~C ⊃ G)

/:. 3. (R v G)

Argument form:

1. (p ⊃ q)
2. (~p ⊃ r)

/:. 3. (q v r)

The standard procedure is using **p to represent the first variable**, **q to represent the second one**, and so on.

Truth Tables for Validity: The General Procedure

1) Write all the premises and conclusion of the argument, horizontally.

1. (p ⊃ q)
2. (~p ⊃ r)

/∴ 3. (q v r)

(p ⊃ q)	(~p ⊃ r)	/∴ (q v r)

Truth Tables for Validity: The General Procedure

2) List all the variables that occur in the formulas (top left, in alphabetical order).

1. (p ⊃ q)
2. (~p ⊃ r)

/:. 3. (q v r)

p	q	r	(p ⊃ q)	(~p ⊃ r)	/:. (q v r)

Truth Tables for Validity: The General Procedure

3) Construct the **base column**: the list of all possible combinations of truth values for the variables.
   - The number of rows in your column will be 2^n (n = the number of variables; 2 = truth values)

1. (p ⊃ q)
2. (~p ⊃ r)

/:. 3. (q v r)

p	q	r	(p ⊃ q)	(~p ⊃ r)	/:. (q v r)
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

Truth Tables for Validity: The General Procedure

4) Compute the truth values for every row in the table.

1. (p ⊃ q)
2. (~p ⊃ r)

/:. 3. (q v r)

p	q	r	(p ⊃ q)	(~p ⊃ r)	/:. (q v r)
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	F	F

Truth Tables for Validity: The General Procedure

5) Look for counterexamples.

1. (p ⊃ q)
2. (~p ⊃ r)

/:. 3. (q v r)

p	q	r	(p ⊃ q)	(~p ⊃ r)	/:. (q v r)
T	T	T	**T**	**T**	**T**
T	T	F	**T**	**T**	**T**
T	F	T	F	T	T
T	F	F	**F**	**T**	**F**
F	T	T	**T**	**T**	**T**
F	T	F	T	F	T
F	F	T	**T**	**T**	**T**
F	F	F	**T**	**F**	**F**

The Partial Method

- The partial method is way to test the validity of an argument **without computing all the rows**.

- This method consists of three steps:

1) Construct your base column.
2) Compute the **truth values of the conclusion column.**
3) Compute only the truth values of the rows in which **conclusion is false**, to look for counterexamples.

- In some cases it is faster to compute only the truth values of the **rows where all premises are true** instead.

The Partial Method: Example

1. (p ⊃ q)
2. (~p ⊃ r)
/:. 3. (q v r)

p	q	r	(p ⊃ q)	(~p ⊃ r)	/:. (q v r)
T	T	T			**T**
T	T	F			**T**
T	F	T			T
T	F	F	**F**	**T**	**F**
F	T	T			**T**
F	T	F			T
F	F	T			**T**
F	F	F	**T**	**F**	**F**

The Partial Method: Beginning by the Premises

1. (p ⊃ q)
2. (~p ⊃ r)
/:. 3. (q v r)

p	q	r	(p ⊃ q)	(~p ⊃ r)	/:. (q v r)
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	
T	F	F	F	T	
F	T	T	T	T	T
F	T	F	T	F	
F	F	T	T	T	T
F	F	F	T	F	

The Short Method

- The short method consists in proving that an argument form is invalid, **by constructing a counterexample directly.**

- It has one single step:

1) **Assign truth values** to your variables that will make the **premises true**, and the **conclusion false**.

- If it is impossible to do this, the argument is valid.

The Short Method

- The short method can also be used **to show validity**, in this way:

1) Look for all the possible assignments of values on which the **conclusion comes up false**

2) Show that, with these assignments of values, **one of the premises must come up false**.