UNIT 15
Proofs in Predicate Logic

- The four quantifier rules
- Symbolization of full arguments
- Proofs for arguments

Quantifier Rules: An Overview

- Quantifier rules allow you to construct proofs with quantified formulas.

- These rules will interact with the basic inference rules (unit 7), the replacement rules (unit 8), and the negation equivalences for quantified formulas (units 11 and 12).

(See slide 11, ppt. 11/ Klenk p. 221, and slide 13, ppt. 12/Klenk, p. 238)

- **There are four quantifier rules**.

Quantifier Rules: An Overview

There are **two types** of quantifier rules: rules for **dropping a quantifier**, and rules for **introducing a quantifier**.

	For dropping the quantifier:	For introducing the quantifier:
Universal	Universal Instantiation (U.I.)	Universal Generalization* (U.G.)
Existential	Existential Instantiation* (E.I.)	Existential Generalization (E.G.)

The rules of Existential Instantiation and Universal Generalization come with some restrictions.

Universal Instantiation (U.I.)

Universal Instantiation (U.I.) is used to **drop the universal quantifier**.

Given a universal formula, any instance of that formula may be inferred.

To apply U.I.: (1) drop the quantifier, and (2) replace the variables in the formula with an individual constant.

Example:

$(x)\ \Phi x$	$(x)\ (Px \supset Mx)$	All U.S. presidents have been men.
________	_____________	
$/\therefore\ \Phi a$	$/\therefore\ (Pc \supset Mc)$	If Coolidge is a president, then he is a man.

In this example, $\Phi x = (Px \supset Mx)$

You can use U. I. for any individual constant, but you should replace all the relevant variables with the same individual constant.

Existential Generalization (E.G.)

Existential Generalization (E.G.) is used to **introduce the existential quantifier**.

Given any formula with individual constants, an existential formula may be inferred.

To apply U.I.: (1) write the existential quantifier to the left of the formula, and (2) replace the constants in the formula with the same variable in the quantifier.

Example:

Φa	Hm • Tm	Mary is honest and trustworthy
/:. (∃x) Φx	/:. (∃x) (Hx • Tx)	Someone is honest and trustworthy

In this example, Φa= (Hm • Tm)

You should replace all the occurrences of m for the variable that is in the quantifier.

A Simple Proof with U.I. and E.G.

1. (x) Ux	Everything is useful.
2. (x) (Ux ⊃ Gx)	Anything that is useful is good.
/:. (∃x) Gx	Something is good

1. (x) Ux	Pr.	
2. (x) (Ux ⊃ Gx)	Pr.	
3. Ua	U.I. 1	--a is useful
4. Ua ⊃ Ga	U.I. 2	--if a is useful, then a is good
5. Ga	M.P. 3, 4	--a is good
6. (∃x) Gx	E. G. 5.	

Existential Instantiation (E. I.)

Existential Instantiation (E. I.) is used **to drop the existential quantifier**.

Given an existential formula, **certain instances may be inferred**.

Restriction: **The individual constant introduced must be new to the proof**.

		Example:	
(∃x) Φx		(∃x) Lx	Someone is lying
________		_______	
/:. Φa	**a is flagged**	/:. La	a is lying

The individual constant used in E.I. **must be flagged**, or marked off.

In the example above, we use "a", supposing that "a" **does not appear anywhere** in the premises, in the conclusion, or in any previous step of the proof.

Hence, to apply E.I.: (1) drop the quantifier, (2) replace the relevant variables with a constant that is new to the proof, and (3) flag the constant.

A Proof with U.I., E.G., and E. I.

1. $(\exists x)\ (Fx \bullet \sim Gx)$	Some fragile things are not made of glass
2. $(x)\ (Wx \supset Gx)$	All windows are made of glass
/ :. $(\exists x)\ (Fx \bullet \sim Wx)$	Some fragile things are not windows

1. $(\exists x)\ (Fx \bullet \sim Gx)$	Pr.
2. $(x)\ (Wx \supset Gx)$	Pr.
3. $Fa \bullet \sim Ga$	E.I., 1 (**flag a**)
4. $Wa \supset Ga$	U.I., 2
5. Fa	Simp., 3
6. $\sim Ga$	Simp., 3
7. $\sim Wa$	M.T., 4, 6.
8. $Fa \bullet \sim Wa$	Conj., 5, 7
9. $(\exists x)\ (Fx \bullet \sim Wx)$	E.G., 8.

A Useful Point for Using E.I.

- **Always use E.I. before using U.I.**

If you use E.I. first, you can use the same individual constant when you use U.I. later.

But if you use U.I. first, you'll have to pick a different individual constant when you use E.I. later.

- Since **U.I. and E.G. are rules without restrictions**, you may use constants that have already appeared in the proof to apply these rules.

Universal Generalization (U.G.)

Universal Generalization (U.G.) is used to **introduce the universal quantifier**.

Given certain instances, an universal formula might be inferred.

Restriction: The instance for U.G. has to be derived **with a subproof** that goes from a constant new to the proof, to the relevant instance:

--> **flag a** **--a is new to the proof.**

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Φa

(x) Φx

To apply U.G.: (1) introduce a constant that is new to the proof, and flag it, (2) derive the relevant instance with that constant, and (3) substitute all the occurrences of the constant with the relevant variable, and write the universal quantifier to the left.

A Proof with U.G.

1. (x) (Cx ⊃ Mx)	All cats are mammals
2. (x) (Mx ⊃ Vx)	All mammals are vertebrates
/:. (x) (Cx ⊃ Vx)	All cats are vertebrates

1. (x) (Cx ⊃ Mx)	Pr.
2. (x) (Mx ⊃ Vx)	Pr.
3. **--> a**	**flagged for U.G.**
4. Ca ⊃ Ma	U.I., 1
5. Ma ⊃ Va	U.I., 2
6. Ca ⊃ Va	H.S., 4, 5.
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7. (x) (Cx ⊃ Vx)	U.G. 6.