Unit 3: Computing Truth Values

- TRUTH TABLES FOR THE BASIC FIVE SENTENTIAL OPERATORS
- TRUTH VALUE COMPUTATION
- TRUTH FUNCTIONAL VS NON TRUTH FUNCTIONAL OPERATORS

Sentential Operators: Quick review

- **Sentential operators** are expressions that are used to construct compound formulas.

- There are five basic sentential operators:

  1) Conjunction
  2) Disjunction
  3) Conditional
  4) Biconditional
  5) Negation

- Every formula in sentential logic has a **truth value**: it is either **true or false**.

Truth Functional Operators

- An **operator** is **truth functional** when the truth value of the compound sentence it forms can be determined just in terms of **the truth values of the component sentences,** plus the **rules of computation** for its operators**.**

Example:

Truth value of (A v B) = Truth value of A

Truth value of B

Computation rule for disjunction

- A **system of logic** is truth functional if and only if **each of its operators is truth functional**.

Truth Tables for Sentential Operators

- Each sentential operator has its own computation rule to determine the truth value of the compound sentence it forms.

- A useful device for representing these computations rules **are truth tables**.

- (Truth tables have other uses, which will be reviewed in chapters 5 and 6)

Truth Tables for Sentential Operators

- A **truth table** is a systematical list of **all the possible combinations of truth values** for the components of a formula.

It gives us the resulting truth value for the whole formula in each of those possible combinations.

- Example:

((A ⊃ B) • (A v B))

A	B	(A⊃B)	(A v B)	(A⊃B) • (A v B)
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

Conjunction (•, "and")

Rule: A conjunction is **true only when both conjuncts are true**.

Otherwise it is false.

P	Q	(P • Q)
T	T	T
T	F	F
F	T	F
F	F	F

Another way of thinking of this rule is:

If **at least one of the conjuncts is false**, the conjunction is **false**.

Disjunction (v, "or")

Rule: A disjunction is **true when at least one of its disjuncts is true**.

Otherwise it is false.

P	Q	(P v Q)
T	T	T
T	F	T
F	T	T
F	**F**	**F**

Another way of thinking of this rule is:

Disjunction is **only false when both disjuncts are false**.

Negation (~, "not")

Rule: Negation **reverses the truth value of the negated formula**.

Hence, a negation is true if the negated formula is false, and false if the negated formula is true.

P	~ P
T	F
F	T

~ ~P
T
F

Biconditional (≡, "if and only if")

Rule: A biconditional is **true when both of its components have the same truth value**.

It is **false** when its components have **different truth values**.

P	Q	(P ≡ Q)
T	T	T
T	F	F
F	T	F
F	F	T

Conditional (⊃, "if, then")

Rule: A conditional is **true** when it has a **true consequent, or a false antecedent**.

P	Q	P ⊃ Q
T	T	T
T	F	F
F	T	T
F	F	T

Another way of stating the rule is:

A conditional is only **false** when the **antecedent is true and the consequent is false**.

Otherwise it is true.

The Conditional: Some Oddities

- The conditional operator is a technical term. It is sometimes referred to as **material conditional**.
- As a technical term, it does not capture the ordinary English use of the particles "if…, then...".
- It establishes **minimal conditions for implication**: in no case the antecedent is true but the consequent is false.
- The **meaning of the sentences** plugged in in the antecedent and consequent is **irrelevant** to the truth of the conditional.

Examples:

"**If** 2 + 2 = 5, **then** I have been to Venus"

F F False antecedent and consequent = True conditional

"**If** 2 + 2 = 5, **then** 2016 was a leap year"

F T False antecedent = True conditional

Non-truth-functional Operators

- An operator is **non-truth functional** if it is **not possible** to determine the truth value of the compound sentence from the truth values of its components.

Examples:

- Epistemic operators: "John believes that ____"
- Modal operators: "It is possible that ____"; "it is necessary that ___"
- Plugging in sentences with the same truth values do not necessarily preserve the truth value of the compound (example on next slide):

Non-truth-functional Operators: Example

A = President Frenk stepped in in 2015 = T

B = Sebastian the ibis came to life in 1926 = T

John believes that president Frenk stepped in in 2015 = T

- but still

John believes that Sebastian the ibis came to life in 1926 = F

- If "... believes that..." was a truth-functional operator, replacing sentence A with sentence B **would result in a true sentence**.

- More generally: Replacing a truth-functional sentence's component with another formula with the same truth value will always result in a **preservation of the original truth value** of the compound sentence.