UNIT 7
THE PROOF METHOD:
EIGHT BASIC INFERENCE RULES

Statement forms and statement instances

The structure of the proof process

Eight basic inference rules

STATEMENT FORMS AND STATEMENT INSTANCES

- **Statement form**: The structure of a sentence. A formula that has sentential variables as its smallest unit.
- **Sentential variables**: Letters that might stand for any value

- **Statement instance:** A formula that has sentential constants as its smallest unit.
- **Sentential constants**: Letters that have a definite, particular value

- Statement forms have **substitution instances** when its variables are replaced by:
- 1) a simple formula (a single constant letter)
- 2) complex formulas

THE STRUCTURE OF THE PROOF PROCESS

- **Proof**: A chain of reasoning which starts with some premises and deduces the conclusion through a series of intermediate steps.

- Every step taken is recorded and numbered.

- Every new step follows from the previous steps, according to **certain specified rules of inference**.

- **Rule of inference**: A basic pattern of reasoning, according to which a conclusion of a certain form may be inferred from premises of a certain form.

EIGHT BASIC INFERENCE RULES

- **Rules for the conditional:**
- Modus Ponens (M.P.)
- Modus Tollens (M.T.)
- Hypothetical Syllogism (H.S.)

- **Rules for conjunction:**
- Simplification (Simp.)
- Conjunction (Conj.)

- **Rules for disjunction:**
- Disjunctive Syllogism (D.S.)
- Addition (Add.)

- **Combined:**
- Dilemma (Dil.)

RULES FOR THE CONDITIONAL:

MODUS PONENS (M.P.)

- If you have a conditional and **the antecedent** of the conditional, you can infer **the consequent** of the conditional

- 1. $p \supset q$
- 2. p

/:. 3. q M. P. 1, 2

RULES FOR THE CONDITIONAL:

MODUS TOLLENS (M.T.)

- If you have a conditional and the **negation of the consequent** of the conditional, you can infer the **negation of the antecedent**.

- 1. $p \supset q$
- 2. $\sim q$
- ___________
- /:. 3. $\sim p$ M.T. 1, 2
-

RULES FOR THE CONDITIONAL:

HYPOTHETICAL SYLLOGISM (H.S.)

- If you have **two conditionals**, and **the consequent of the first one is the antecedent of the second one**, you can infer a conditional that has the antecedent of the first conditional as antecedent, and the consequent of the second conditional as consequent.

- 1. $p \supset q$
- 2. $q \supset r$
- __________
- /:. 3. $p \supset r$ H. S. 1, 2

RULES FOR CONJUNCTION:

SIMPLIFICATION (SIMP.)

- If you have a conjunction, you can infer **either of its conjuncts**.

- This rule has two forms:

- 1. p • q
- _______
- /:. 2. p Simp. 1

1. p • q

____

/:. 2. q Simp. 1

RULES FOR CONJUNCTION:

CONJUNCTION (CONJ.)

- For **any two statements** you have, you can infer the conjunction of those statements.

- 1. p
- 2. q
- ______
- /:. p • q   Conj. 1, 2.

RULES FOR DISJUNCTION:

DISJUNCTIVE SYLLOGISM (D.S.)

- If you have a disjunction and **one of the disjuncts is negated**, you can infer the other disjunct.

- This rule has two forms:

1. p v q	1. p v q
2. ~ p	2. ~ q
________	_________
/:. 3. q D.S. 1, 2.	/:. p D.S.1, 2.

RULES FOR DISJUNCTION:

ADDITION (ADD.)

- For **any statement you have**, you can construct a disjunction **with any other statement**.

- 1. p
- _____
- /:. p v q   Add. 1

DILEMMA (DIL.)

- If you have **two conditionals** and **a disjunction formed with the antecedents** of the conditionals, you can infer **a disjunction formed with the consequents** of the conditionals.

- 1. $p \supset q$
- 2. $r \supset s$
- 3. $p \vee r$
- ________
- /∴ 4. $q \vee s$ Dil.1, 2, 3.